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DISCUSSION

Rejoinder on: Extensive facility location problems on networks: an updated review

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We are grateful to the discussants of this survey for providing thoughtful comments and interesting suggestions. We are pleased to note that all of them found the survey complete, well structured, and useful to provide a clear overview on the topic of location of extensive facilities on graphs. They also recognized the potential of this article to initiate and encourage further research in this area.

First of all, we thank the discussants for their careful reading that led to fix some technicalities and missing details that improved the quality of the original manuscript. On top of that, we would like to comment on some specific remarks and suggestions raised in the discussion.

Wang pointed out some improved complexity results related to the p-core problem, $p \geq 2$ Novik (1996), the k-tree center problem and the minmax regret path location problem that have been included in the revised version of the paper.

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Wang also suggested introducing two additional general tools useful in handling problems in this area, namely, (1) the formula for the efficient computation of the maximum weighted distance from a vertex v in a rooted tree to the vertices of the subtree rooted at one of its children u, and (2) the notion of bisector of a pair of vertices. We have included these new elements in Section 3 of our paper devoted to *General Techniques*.

In addition, all discussants made suggestions on possible directions of future research, extensions and further insights on the topic.

Mesa comments on some problems that were not included in this survey for the sake of page limitations, namely the location of extensive facilities in continuous spaces and the location of cycles. We agree on the importance of those problems but, as he himself mentions, we have explicitly excluded them for consideration to have a more focused paper and to limit the size of our manuscript. Moreover, Mesa mentions two additional important problems. The first one involves the coverage of demand points that will deserve further attention and that is so far scarcely considered in the literature, whereas the second one relates to situations where the demand is given by pairs of nodes and subnetworks with specific properties (faster speed or similar) that should be taken into account.

Saldanha points out questions regarding hardness versus applicability of some specific extensive facility location problems. The first issue is related to the location of structures with fixed size. In this regard, relating to the objective functions covered in this paper, the complexity does not change if the size is bounded above or exactly fixed. However, the latter case has not been often analyzed in the literature due to lack of applicability. The second issue is about locating cycles on networks. Here, the complexity is higher and probably applicability is itself limited by the hardness of the problem.

Saldanha also makes interesting considerations about MIP formulations and heuristic algorithms for some of the NP-hard problems that appear in the area. We note that we had already mentioned this kind of approach in Section 5.1 and 5.4 providing some seminal references by Kim et al. (1996), George and Revelle (2003) and Puerto and Tamir (2005). Relying on mathematical programming formulations, one can develop suitable heuristic algorithms for NP-hard problems. Some contributions on this direction have been already done, as for instance by Lari et al. (2008).

Finally, Saldanha suggests further insights for future research. The first one concerns the multiperiod modeling framework that for extensive facility location has not been explored. This sounds a challenging topic due to the importance of long-lasting strategic decisions in the area of location. Another important aspect mentioned in his report is about uncertainty. In our survey, we give some hints on dealing with uncertainty throughout robust optimization. Modeling uncertainty by probabilistic information is an appealing, alternative approach that, however, may lead to computationally difficult problems.

Also, Wang gives possible directions for future research. He raises the issue of finding lower bounds for the complexity of polynomially solvable problems. In the summary tables at the end of the survey, we already state those problems for which lower bounds or optimal algorithms are known. Focusing on trees, chances are higher of succeeding on this challenging task. Another idea for future development is the

design of efficient approximation algorithms for NP-hard problems on this class. As a starting point of this research, it may be advisable to concentrate on NP-hard problems that appear in our tables.

Finally, we would like to thank again the discussants for their stimulating comments and suggestions, and for their help in filling the gap in our references list and tables. We hope that our survey paper, together with this discussion, could provide useful material and knowledge for researchers involved in extensive facility location on graphs.

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